

## ELASTIC STRENGTH OF HIGH PRESSURE VESSELS WITH A RADIAL CIRCULAR CROSS BORE

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### ABSTRACT

*The purpose of this study was to determine elastic strength of a high pressure vessel with a radial circular cross bore, for various thickness ratios and cross bore sizes. Seven cross bored cylinders with thickness ratios between 1.4 and 3.0, having both small and large radial circular cross bore were studied. Three dimensional linear finite element analyses were used in this work. It was found that unlike in plain cylinders, the difference in the magnitude of working stresses between the Von Mises' and Tresca's theories in cylinders with radial circular cross bore was insignificant. Besides, the effects of cross bore size and thickness ratio on shearing and working stresses were observed to exhibit similar stress distribution patterns. The magnitude of the working stress in the cylinder increased with the cross bore size, reaching an overall stress factor range of 2.5 to 7.07.*

**KEYWORDS:** Stress Concentration Factor, High Pressure Vessel, Cross Bore & Elastic Strength

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### 1. INTRODUCTION

Thick cylinders with end enclosures are used as high pressure vessels to store large amounts of fluids at extreme temperatures and pressures [1, 2]. Pressure vessels have various applications in industries such as in petrochemicals, pharmaceuticals and thermal and nuclear power plants, among others [3-4]. Therefore, their importance in our daily lives cannot be overemphasized.

Pressure vessels are constructed with side openings in their walls [5]. The openings provide provision for fitting essential operation and instrumentation accessories. These accessories include relief and safety valves, gas inlets, flow circuits meters, inspection manholes, lubrication etc. [2]. As a result, openings in the design of pressure vessels are inevitable.

Various terminologies are used to describe openings in a pressure vessel. Aside hole refers to a single hole in one side of the vessel [6-7]. Whereas, a cross bore refers to a transverse hole in both sides of the vessel. Whenever an opening is constructed at centroidal axis of the vessel, it is termed as a radial cross bore. On the other hand, when the same opening is placed in any other chord away from the central axis is referred to be an offset cross bore [7]. Moreover, cross bores with different sizes and shapes are used in the design of pressure vessels. The size of the cross bore range from small drain pipe to large manhole [3]. Besides, only circular and elliptical cross bore shapes are the commonly used [2, 8].

Drilling of cross bores on the wall of the pressure vessel create geometric discontinuity. This discontinuity results to change in stress distribution pattern close to their vicinity. Geometric discontinuity acts as stress raisers, thus creating regions of high stress concentration especially at the vicinity of the discontinuity [9].

Unfortunately, the elemental stress equations such as the Lamé's theory cease to apply at these regions with high stress concentration [3].

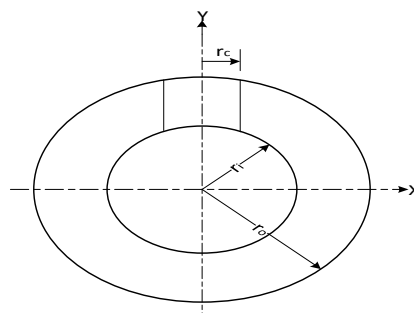
Stress concentration at these regions is calculated by a dimensionless factor called Stress Concentration Factor (SCF) [3]. Theoretical SCF is defined as the ratio of maximum stress at a particular point due to the introduction of a discontinuity and the corresponding stress occurring at the same point when there is no discontinuity [10]. SCF is determined for various stress criteria such as maximum tensile stress (hoop), Von Mises or Tresca. The choice to use a particular criterion depends on the desired working conditions of the vessel [10].

High magnitudes of SCF are associated with problems encountered in the design and operation of pressure vessels such as fractures, fatigue and local yielding. Generally, fatigue failures and cracks initiate at points of high stress concentration. Regrettably, once a crack is initiated, it becomes itself an intense stress concentrator. According to Cole *et al.* [11], high magnitudes of SCF act as points of weakness leading to reduction in the vessel strength as well as its fatigue life. This consequently may reduce the pressure carrying capacity of the pressure vessel by up to 60 % [12] when compared to a plain vessel without cross bores. Approximately, 24.4% of the total number of accidents in industrial processes is due to failures associated with pressure vessels [13]. These failures are catastrophic have led to loss of human life, damage of property, displacements of persons and environmental pollutions [13]. Therefore, SCF is important parameters to be considered in the vessel design since it enables effective comparison of stresses between different parameters regardless of their size, shape, thickness or the applied load.

From the preceding paragraph, there is need for pressure vessel designers to establish the elastic strength of cross bored pressure vessels. This leads to precise determination of safe working pressures and safety factors in the design of pressure vessels. The aim of this study was to determine elastic strength of a high pressure vessel with a radial circular cross bore, for various thickness ratios and cross bore sizes.

## 2. METHODOLOGY

Seven cross bored cylinders with thickness ratios ( $K$ ) of 1.4, 1.5, 1.75, 2.0, 2.25, 2.5 and 3.0, having both small and large radial circular cross bore were studied. These cylinders ( $K$ ) were selected to coincide with those discussed in the reviewed literature by references [7, 14-15]. A total of five different radial circular cross bore sizes were investigated in each cylinder. Circular cross bore sizes with size ratios (cross bore to main bore ratio) of 0.1, 0.3 and 0.5 were classified as small [16]. Whereas, those with cross bore size ratios of 0.7 and 1.0 were categorized as large. A configuration showing a radial cross bore in thick walled cylinder is shown in Figure 1.



**Figure 1: Cross Bore Configuration**

Where,

$r_c$  is the radius of the cross bore

$r_i$  is internal radius of the main bore

$r_o$  is the external radius of the main bore

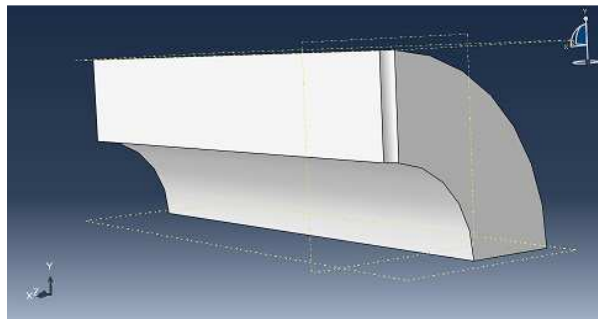
## 2.1 Finite Element Analysis

Three dimensional linear finite element modelling analyses were performed on the high pressure vessels with radial cross bores using a commercial software program called Abaqus version 6.16. A total of 35 different part models were created and analysed. Owing to the symmetrical configuration of the cylindrical vessel, only an eighth profile of the structure was used.

The following standard procedure used in Abaqus modelling software was followed;

### 2.1.1 Creation of a Model

A three dimensional deformable solid body was created by drawing an eighth profile of the front elevation structure of the cylinder. The created elevation structure was then extruded to form a suitable depth of the cylinder. The depth of the cylinder was equal to three times the cylinder's external diameter. This depth was long enough to restrict the effects of the closed ends' closures of the cylinder vessels from being transmitted to the other far end of the cylinder. The radial cross bore was then formed at the creation step using cut extrude technique while applying full boundary drawing constraints. One of the model profiles created at this stage is shown in Figure 2;



**Figure 2: Selected Profile of the Model**

### 2.1.2 Creation of Material Definition

In this study, a linear elastic model was adopted. The following material properties were used; Young modulus of elasticity 190 GPa, Poisson's ratio 0.3 and density 7800 Kg/m<sup>3</sup>. The material properties chosen for this simulation were similar to those reported in references [17, 18].

#### 2.1.2.1 Assigning of Section Properties and Model Assembly

The section properties of the whole model profile were defined as being solid and homogenous. After which creation of a single assembly was done. These preceding procedures allowed the creation of a part instance that was independent of the mesh. The profile of the model, indicated in Figure 2, was then oriented to conform to the global Cartesian co-ordinates axes i. e. X, Y and Z axes.

### 2.1.2.2 Analysis Configuration

The analysis used for this modelling was configured by creating a static pressure step. It is important to mention that the application of different types of loads and boundary conditions are interlinked with each analysis steps.

### 2.1.2.3 Application of Boundary Conditions

Symmetry boundary conditions were then applied at each of the three cut sections of an eighth profile of the cylinder. These symmetry boundary conditions were applied at cut regions in X, Y and Z axes thus preventing any rigidity movement.

### 2.1.3 Application of loads

The model was then loaded with an internal pressure at both the main bore and the cross bore. The internal pressure was taken as 1MPa [19] in line with the standard practice used in pressure vessel analyses. In addition, a uniform axial stress  $\sigma_z$ , was computed using equation (1) for each cylinder. The magnitudes of axial stress calculated were then applied at the far end of each corresponding model to simulate the end effects generated by the closed end closures in the pressure vessels.

$$\sigma_z = \frac{p_i}{K^2 - 1} \quad (1)$$

Where

$P_i$  is the internal pressure.

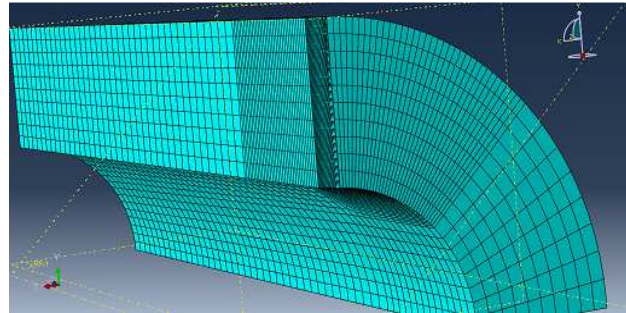
K is the thickness ratio

### 2.1.4 Meshing of the Model

The local mesh refinement of the model was done using a combination of both the H-element and the P-element techniques. The H-element refinement technique was achieved through two stages. In the first stage, the model was divided into small geometrical sections. In the second stage, the mesh around the cross bore region was biased by increasing the number of elements, commonly referred to as mesh density. In fact, the mesh was found to converge when the size of element was between 0.003 m and 0.004 m. This high mesh density around the cross bore region increased the capture of the localised stress concentration [20]. This approach gives results with a high level of accuracy without significantly increasing the computer run time [20].

Alternatively, the P-refinement technique approach, which depends on the degree of polynomial, was achieved by use of the second order differential equations with reduced integration. The mesh verification was carried out to establish the element quality and identify any distorted elements. Generally, element distortion leads to erroneous results. Thus, to eliminate the occurrence of element distortion, the percentage tolerance for both the element warnings and errors were kept at zero. In addition, the choice of element used for this modelling, was made carefully so as not to introduce element distortions. In this regards, therefore, 20-noded second order, C3D20R hexahedral (brick) isoparametric elements were used in cross bore size ratios of 0.1, 0.3, 0.5, and 0.7. On the other hand, second order C3D10 tetrahedral elements with 10 nodes were used for pipe junction models.

It is important to note that only second order hexahedral and tetrahedral elements are recommended for stress concentration problems when using Abaqus software. Usually, hexahedral elements give results with a high degree of accuracy [20] compared to other elements, while, tetrahedral elements are less sensitive to the initial shape of the element. Hence, their rate of vulnerability to distortion is low. A meshed profile of one of the model parts is shown in Figure 3.



**Figure 3: Selected Profile of the Mesh**

### 2.1.5 Validation of the Model

The accuracy of the results depends on the mesh quality and its density. The validation of the results as well as the mesh convergence, were done by comparing the FEA displacements and principal stresses (hoop, radial and axial), with their corresponding analytical results in areas far away from the cross bore vicinity [11, 21]. Usually, the effects of geometric discontinuity are limited to its surrounding area. For instance, the effects of a cross bore is limited to its vicinity, approximated to be a linear length of 2.5 cross bore diameters [11, 21]. Besides, results from other similar works in the reviewed literature were also used to validate the developed model further.

### 2.1.6 Stress Concentration Factor

In this work, the stress concentration factor was defined as the ratio of localised critical stresses in a cross bore cylinder to the corresponding one in a similar cylinder without a bore. This definition exemplifies the intensity of stress concentration at each particular point of interest. It is worth noting that the peak stresses in cross bored thick cylinders do not necessarily occur at the intersection of the main bore and cross bore [11]. Therefore, utmost care should be taken when defining the SCF.

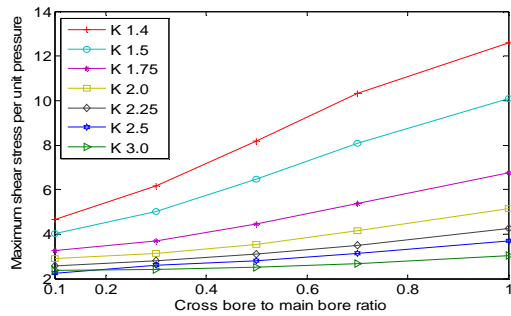
In most practical engineering applications, the design strength of a component is based on the peak stresses. Besides, cracks and fatigue failure initiate from regions of high stress concentration [10]. Therefore, the SCF for cylinders having different cross bore sizes and thickness ratios were calculated based on locations with the highest magnitudes of hoop stress in the cylinder.

In this article, only shearing and working stresses with have adverse effects on elastic strength of the vessel were discussed. Other types of stresses generated from this simulation are presented elsewhere.

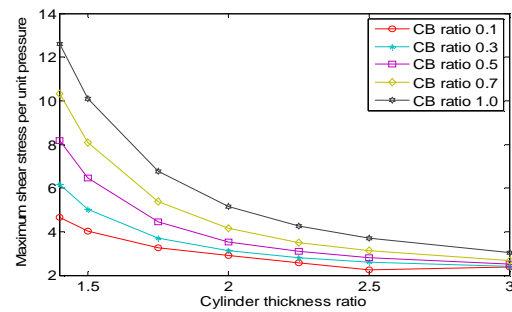
## 3. RESULTS AND DISCUSSIONS

### 3.1 Effects of Cross Bore Size and Thickness Ratio on Maximum Shearing Stress

The magnitude of maximum shear stress depends on the difference between the hoop and radial stresses. Since the hoop stress depends on the thickness ratio, a direct comparison among the thickness ratios is not possible. The maximum shearing stress was found to be affected by both the cross bore size and thickness ratio as illustrated in Figures 4 and 5.



**Figure 4: Maximum Shear Stress vs Cross Bore Size**



**Figure 5: Maximum Shear Stress vs Thickness Ratio**

The maximum shear stress increased with an increase in cross bore size. Moreover, it was also observed to reduce with increase in the thickness ratio. This occurrence was attributed to the varying magnitude of the hoop stress along the cross bore. Since the corresponding radial stress along the cross bore is constant.

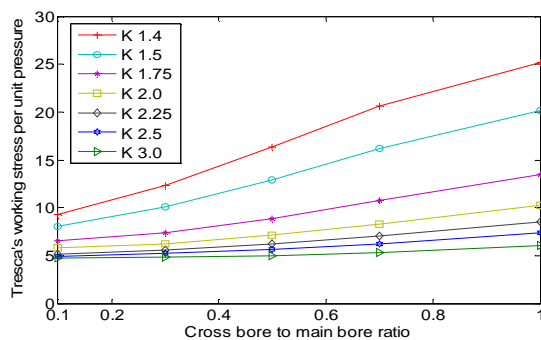
Generally, the highest magnitude of shear stress was observed to occur in RZ plane of the cylinder in the radial circular cross bores.

### 3.1.1 Elastic Failure Theories

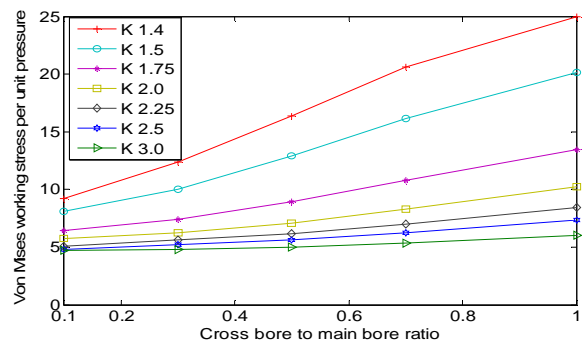
In this section, the working stresses together with their corresponding stress concentration factors were computed using elastic failure theories applicable to ductile materials, namely the Tresca's and the Von Mises's Theories. The results obtained were then evaluated to establish their effects on cross bore size in addition to the thickness ratio.

### 3.2 Effects of the Cross Bore Size and Cylinder Thickness Ratio on Elastic Working Stress

A similar stress distribution pattern exhibited in the shear stresses was also displayed in this section as shown in Figures 6 to 9.



**Figure 6: Tresca's Stress vs Cross Bore Size**



**Figure 7: Von Mises's Stress vs Cross Bore Size**

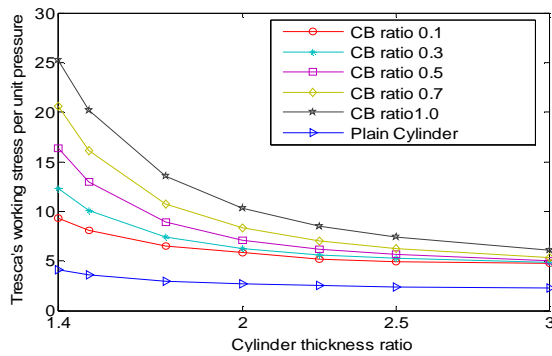


Figure 8: Tresca's Stress vs Thickness Ratio

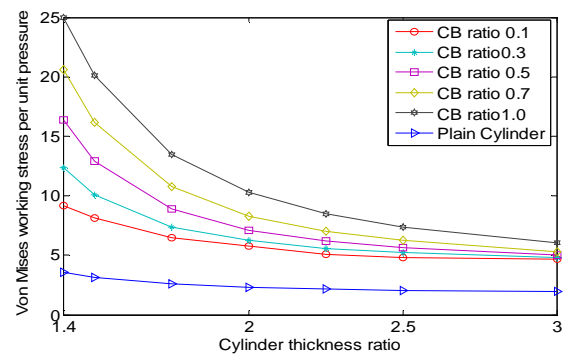


Figure 9: Von Mises's Stress vs Thickness Ratio

Usually, the difference between the working stress of the Tresca's and the Von Mises's Theories in plain cylinders without cross bore is approximately 15.5%. However, after the introduction of the cross bore, it was seen that the difference in the magnitude resulting from the two theories reduced tremendously. For instance, for  $K=1.4$ , the differences in working stress magnitude between the two failure theories due to the introduction of cross bore size ratios of 0.1, 0.3, 0.5, 0.7 and 1.0 were 1.13%, 0.08%, 0.06%, 0.1% and 0.92%, respectively. A similar trend was also replicated in  $K=3$  where the working stress differences for the same cross bore ratio sizes discussed previously were 1.32%, 0.2%, 0.16%, 0.21% and 0.43%, respectively. This occurrence was attributed to small magnitudes of axial stresses along the cross bore surface. In fact, the analytical solution derived by Ford and Alexander [10], assumed a zero magnitude of axial stress along the cross bore. This finding was contrary to that of plain cylinders where the axial stress is constant across the thickness and its magnitude is relatively high.

Further comparison between cross bored cylinders and plain cylinders revealed higher working stresses in the latter, as illustrated in Figures 8 and 9.

### 3.3 Effects of Cross Bore Size and Cylinder Thickness Ratio on Elastic Working Stress Concentration Factor

The variation of elastic working stress concentration factor with cross bore size and thickness ratio are illustrated in Figures 10 to 13.

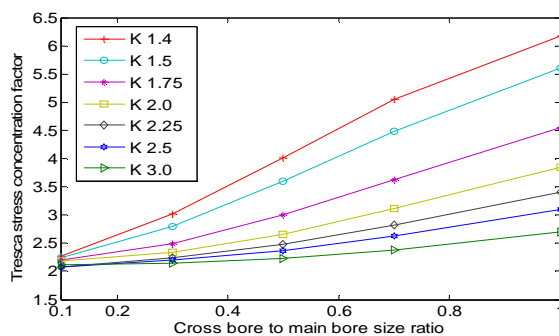


Figure 10: Tresca's SCF vs Cross Bore Size

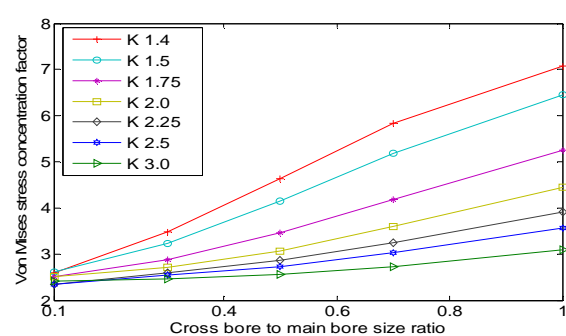


Figure 11: Von Mises's SCF vs Cross Bore Size



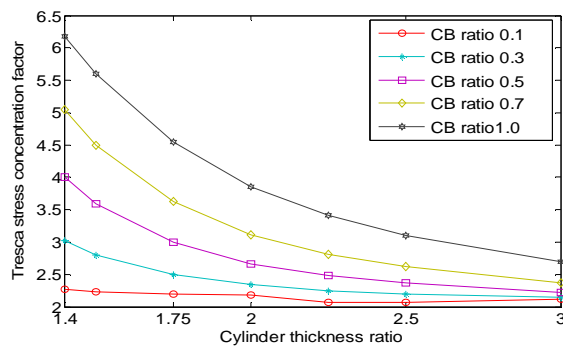


Figure 12: Tresca's SCF vs Thickness Ratio

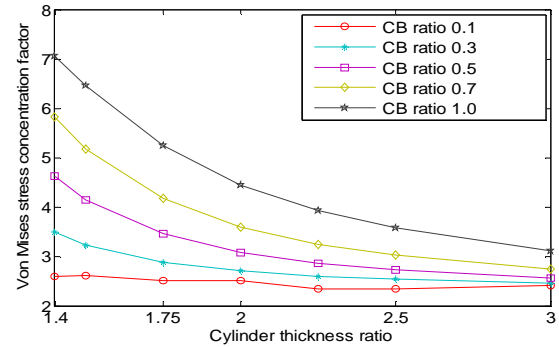


Figure 13: Von Mises's SCF vs Thickness Ratio

It was observed that as the cross bore size ratio increased from 0.1 to 1.0, the maximum working stresses predicted by Tresca's and Von Mises theories increased by a factor of approximately 2.5 to 7.07, in  $K = 1.4$ . Likewise, within a similar range, the minimum increase in working stresses was recorded in  $K = 3$  where the working stress factor predicted by the two theories increased from 2.1 to 3.1. Therefore, for a pressure vessel to operate under elastic stress conditions the internal pressure would be reduced by a similar corresponding factor with regard to the choice of thickness ratio and the cross bore size.

From the preceding sections, it is evident that the stress variation emanating from the elastic working stress and the maximum shear stress exhibited a close resemblance. This occurrence was attributed to the fact that these stress theories are dependent on the three principal stresses, namely hoop, radial and axial. However, only the maximum principal stress (hoop) had a major effect on the overall stress along the cross bore depth due to its high magnitude. This is because, the magnitude of the radial stress also known as gauge pressure, is constant along the surface of the cross bore. Whereas, the magnitude of axial stress on the cross bore surface is small.

#### 4. CONCLUSIONS

- The difference in the working stress magnitude between the Von Mises' and Tresca's theories along a radial circular cross bore was insignificant. Unlike that of a plain cylinder without a cross bore which is constant at 15.5%.
- The effects of cross bore size and thickness ratio on shearing and working stresses exhibited similar stress distribution patterns.
- Introduction of a radial circular cross bore increases the magnitude of the working stress. The maximum working stress predicted by Von Mises' and Tresca's theories in a cylinder with a radial circular cross bore increased by a stress factor ranging from 2.5 to 7.07.

#### List of Abbreviations

**K** - Thickness ratio (outer diameter to inner diameter)

**SCF** - Stress Concentration Factor



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## REFERENCES

1. Hyder JM, Asif M (2008) Optimization of location and size of opening in a pressure vessel cylinder using ANSYS. *Eng. Fail. Anal.* 15: 1-19.
2. Kihui JM, Masu LM (1995) The effect of chamfer and size on the stress distributions in thick-walled cylinder with a cross bore under internal pressure. *R & D journal* 73-78.
3. Kharat A, Kulkarni V (2013) Stress concentration at openings in pressure vessels- A review. *Int J Innov Res Sci Eng Technol* 2: 3.
4. Jeyakumar M, Christopher T (2013) Influence of residual stresses on failure pressure of cylindrical pressure vessels. *Chin. J. Aeronaut.* 26: 1415-1421.
5. Masu LM, (1998) Numerical analysis of cylinders containing circular offset cross bores. *Int. J. Press. Vessels Pip.* 75: 191-196.
6. Peters DT (2003) Effect of blend radius on stress concentration factor of crossbored holes in thick walled pressure vessels. *Conference proceeding on high pressure technology for the future, Cleveland, Ohio, USA, 20-24 July 2003, pp. 53-57.*
7. Makulsawatudom P, Mackenzie D, Hamilton R (2004) Stress concentration at crossholes in thick cylindrical vessels. *J. Strain Anal. Eng. Des.* 39: 471-481.
8. Nagpal S, Jain N, Sanyal S (2012) Stress concentration and its mitigation techniques in flat plate with singularities- A critical review. *EJ* 16: 1.
9. Ford H, Alexander J (1977) *Advanced mechanics of materials.* John Wiley and sonsinc., Canada, second edition.
10. Cole BN, Craggs G, Ficenec I (1976) Strength of cylinders containing radial or offset cross-bores. *J. Mech. Engng Sci.* 18: 6.
11. Nabhani F, Ladokun T, Askari V (2012) Reduction of stresses in cylindrical pressure vessels using Finite Element Analysis - From Biomedical Applications to Industrial Developments, <http://www.intechopen.com/books/finite-element-analysis-from-biomedical-applications-to-industrial-developments/reduction-of-stresses-in-cylindrical-pressure-vessels> (accessed on 09/03/2014).
12. Deshpande, Neela., Kulkarni, S. S., Pawar, Tejaswinee., & Gunde, Vijay. (2014). Experimental investigation on strength characteristics of concrete using tyre rubber as aggregates in concrete. *International Journal of Applied Engineering Research and Development*, 4(2), 97-108.
13. Nihous GC, Kinoshita CK, Masutani SM (2008) Stress concentration factors for oblique holes in pressurized thick walled cylinders. *J. Pressure Vessel Vessels Technol.* doi: 10.1115/1.2891915.
14. Steele CR, Steele ML, KhathlanA (1986) An efficient computational approach for a large opening in a cylindrical vessel. *J. Pressure Vessel Vessels Technol.* 108: 436-442.
15. Chaudhry V, Kumar A, Ingole SM, Balasubramanian AK, Muktibodh UC (2014) Thermo-mechanical transient analysis of reactor pressure vessel. *Procedia engineering* 86: 809-817.

16. Choudhury A, Mondol SC, Sarkar S (2014) Finite element analysis of thermos mechanical stresses of two layered composite cylindrical pressure vessel. *Int. j. res. appl.* 21(14): 341-349.
17. Gerdeen JC (1972) Analysis of stress concentration in thick cylinders with sideholes and crossholes. *Trans. ASME, Journal Engineering Industry*, 94: 815-823.
18. Sawant, S. M., & Hujare, D. P. (2013). Thermo-mechanical analysis for skirt of pressure vessel using FEA approach. *International Journal Of Mechanical Engineering (IJME)* ISSN, 2319-2240.
19. Fagan MJ (1992) *Finite element analysis theory and practice*. Longman, UK, ISBN 0-470-21817-7.
20. Kihuu JM (2002) Numerical stress characterization in cross bored thick walled cylinders under internal pressure. *PhD thesis, The University of Nairobi*.